



THE STABILITY OF STEADY FILTRATION FLOWS OF A GAS-CONDENSATE MIXTURE†

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Cylindrically symmetric filtration flows of a two-phase “gas–condensate” type mixture with phase transitions are considered. A sufficient test of absolute stability in the small in the case of steady flows is found subject to certain assumptions, the strongest of which is that there is no mobility threshold for the liquid phase. It is shown, by numerical estimates, that this stability test is satisfied in the case of real filtration flows. © 1997 Elsevier Science Ltd. All rights reserved.

It is well known that single-phase steady filtration flows are stable. In the theory of the filtration of multi-component mixtures with phase transitions, the stability of steady-state flows has not been fully investigated. On the basis of the results of a numerical simulation, it has been reported [1, 2] that non-decaying oscillatory processes are observed in the case of the filtration of a gas–condensate mixture in the neighbourhood of an operational well. The opinion has arisen in this connection that instability accompanying filtration is possible and, moreover, that it is very common in practice.

It is shown below that, in an extensive class of cases, the steady filtration flows of a gas–condensate mixture are absolutely stable in the small. The question of stability with respect to finite perturbations remains open.

In Section 1 we investigate a number of general properties of flows of a multi-component, two-phase mixture with phase transitions. The dynamics of small perturbations against the background of the exact steady solutions described previously in [3–5] are considered in Section 2 and a sufficient condition for the absolute stability of these solutions is obtained. In Section 3, it is verified that, in typical cases, this criterion is satisfied with a large margin.

1. Consider an $(M + 1)$ -component mixture and suppose n_i the corresponding molar densities of the components. Henceforth, the Latin subscripts i, j, k take values of $0, \dots, M$ and the Greek subscripts and superscripts α and β take values of $1, \dots, M$. Summation over repeated subscripts is assumed everywhere. We shall only study isothermal processes and, for brevity, the temperature dependence of all mechanical and thermodynamic quantities is therefore omitted.

We know from statistical physics [6] that the free energy per unit volume $f = f(n_i)$ is defined for the homogeneous states of a mixture and that this free energy is a smooth and single-valued function of the densities of the components n_i . The thermodynamic relations

$$df = \kappa_i dn_i, \quad f = -p + \kappa_i n_i \quad (1.1)$$

are satisfied, where κ_i are the chemical potentials and p is the pressure. Duhem’s equality

$$dp = n_i d\kappa_i \quad (1.2)$$

follows from (1.1).

Relation (1.1) enables us to calculate the chemical potentials and the pressure in the mixture from the known free energy.

If the function $f = f(n_i)$ is convex, then the homogeneous states of the mixture are thermodynamically stable in bulk. This function is not convex in the case of a two-phase “gas–condensate” type mixture. In the latter case, the thermodynamic stability of a homogeneous state n_i is checked by considering all possible virtual divisions into phases n_{i1}, n_{i2}

$$n_i = sn_{i1} + (1 - s)n_{i2} \quad (1.3)$$

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where s is the volume fraction of phase 1 ($0 \leq s \leq 1$). If the quantity

$$f^* = sf(n_{i1}) + (1-s)f(n_{i2}) \quad (1.4)$$

turns out to be less than $f(n_i)$ then the homogeneous state is thermodynamically unstable and the heterogeneous state (1.3), which ensures a minimum value of the quantity (1.4), is stable.

In the case of two-phase states, we use n_{ic} and n_{ig} to denote the molar densities in the liquid phase (condensate) and the gas phase respectively. We again define the free energy unit per volume $f = f(n_i)$ in the two-phase domain according to formula (1.4) which specifies the value of the free energy in the stable heterogeneous state. Hence, instead of the initial function $f = f(n_i)$, we shall consider its convex envelope, which is denoted by the same symbol. The new free energy is a doubly differentiable function. However, unlike the initial free energy, the second derivatives of $f(n_i)$ can have discontinuities accompanying a transition from a single-phase domain to a two-phase domain.

It can be shown that, with the new definition of the free energy, the thermodynamic relations (1.1) and (1.2) are satisfied as before but, in the two-phase domain, $f = f(n_i)$ and p are now the chemical potentials and pressure in each of the phases.

Since $f = f(n_i)$ is now a convex function, the symmetric matrix

$$\alpha_{ij} = \partial \kappa_i / \partial n_j = \partial^2 f / \partial n_i \partial n_j$$

is non-negative definite. Using this matrix and relation (1.2), it is possible to calculate the derivative of the pressure with respect to the total density $n = n_0 + \dots + n_M$ for a fixed composition $c_i = n_i/n$

$$(\partial p / \partial n)_{c_i} = n \alpha_{ij} c_i c_j$$

This quantity is always non-negative although it can undergo jumps on binodals. The previously discussed possibility of an alternation in the sign of the compressibility in the case of a gas–condensate mixture [1, 2] is therefore excluded.

We will now assume that there is an unsteady, cylindrically symmetric, filtration flow in a homogeneous, isotropic reservoir with a porosity m and permeability k in the neighbourhood of an operating well. Let r be the distance up to the axis of the well. The local conservation laws for the components [7]

$$\partial_t(mn_i) + r^{-1} \partial_r(rJ_i) = 0 \quad (1.5)$$

then hold.

According to Darcy's law, the expressions

$$J_i = -kK_i \partial_r p, \quad K_i = f_g n_{ig} \mu_g^{-1} + f_c n_{ic} \mu_c^{-1} \quad (1.6)$$

hold for the fluxes J_i when capillary forces are neglected.

The quantities n_i , n_{ig} and n_{ic} are related by the equation

$$n_i = sn_{ic} + (1-s)n_{ig}$$

where s is the saturation of the pore space with the liquid phase. In relations (1.6), $\mu_g = \mu_g(n_{ig})$, $\mu_c = \mu_c(n_{ic})$ are the shear viscosities of the gas and the condensate respectively and $f_g = f_g(s)$, $f_c = f_c(s)$ are the phase permeabilities of the gas and the condensate.

Since s , n_{ig} and n_{ic} are thermodynamic functions of the densities n_i , (1.5) is the complete system of equations for the unknown functions $n_i = n_i(t, r)$.

We shall now discuss the boundary conditions. Let r_w be the radius of the well with respect to the drill bit and let r_0 be the radius of the feed contour. We impose the following conditions on the pressure

$$p|_{r=r_w} = p_w, \quad p|_{r=r_0} = p_0, \quad p_w < p_0 \quad (1.7)$$

In the case of gas–condensate deposits, a situation is typical when the mixture in the reservoir is in the gas phase under the initial temperature and pressure conditions. When the reservoir pressure is reduced during the development of the deposit, the phenomenon of so-called retrograde condensation [8] occurs when the gas phase becomes thermodynamically unstable and a liquid condensate separates out. Under the conditions in the reservoir, the condensate occupies a small part of the pore space and,

because of the low values of the phase permeability, it may be considered to be hydrodynamically immobile. The condensate acquires mobility close to operating reservoirs where it can occupy a large part of the pore volume [3–5]. Let c_{i0} be the composition of the mobile (gas) phase of the reservoir mixture which enters into the feed contour of the well. If p_d is the pressure at the start of the condensation for a mixture of this composition then, generally speaking, $p_0 \geq p_d$. In the case when a condensate has separated out in the reservoir, the mobile phase is saturated and $p_0 = p_d$.

We now impose the additional boundary condition

$$c_i|_{r=r_0} = c_{i0} \quad (1.8)$$

and introduce the new quantities

$$n_g = \sum n_{ig}, \quad n_c = \sum n_{ic}$$

$$K = \sum K_i = f_g n_g \mu_g^{-1} + f_c n_c \mu_c^{-1}, \quad C_i = K_i / K$$

(the summation is carried out from $i = 0$ to $i = M$).

In accordance with their definition, the quantities C_i can be interpreted as the concentrations of the components of a certain mixture which separates out under a pressure p into the same phases as the initial mixture. The corresponding total molar density N , partial densities N_i , and liquid phase fraction S are determined from formulae

$$N = K / (f_g \mu_g^{-1} + f_c \mu_c^{-1}), \quad N_i = N C_i = (1 - S) n_{ig} + S n_{ic}$$

$$S = f_c \mu_c^{-1} / (f_g \mu_g^{-1} + f_c \mu_c^{-1}) \quad (1.9)$$

We now consider the inverse problem of determining the parameters of the initial mixture n_i using the known values of C_i and p . Knowing the free energy for the heterogeneous states f , it is possible to determine the quantities n_{ig} , n_{ic} , S using the values of C_i and p . In order to calculate n_i from the known values of n_{ig} , n_{ic} , it is necessary to find the value of s for which Eq. (1.9) holds. The right-hand side of this equation depends monotonically on s . However, if there are mobility thresholds for the phases, this dependence will not be strictly monotonic. In order to ensure the unique solvability of Eq. (1.9) in the class of processes being considered, when a gas phase is necessarily present, we shall assume that there is no mobility threshold for the liquid phase.

In connection with this assumption, we note that, if capillary forces are taken into account, when there is a threshold mobility in the case of the condensate, then the steady-state solution is non-unique [4, 5]. Consequently, in the latter case, any perturbation of the initial conditions which is confined to the class of exact steady-state solutions, does not decay and absolute stability cannot hold.

We now introduce the new coordinate $\eta = \ln(r/r_w)$ and rewrite problem (1.5), (1.7) and (1.8) in the form

$$m \partial_r (r^2 n_i) - k \partial_\eta (K C_i \partial_\eta p) = 0 \quad (1.10)$$

$$p|_{\eta=0} = p_w, \quad p|_{\eta=\zeta} = p_d, \quad C_i|_{\eta=\zeta} = c_{i0} \quad \left(\zeta = \ln \frac{r_0}{r_w} \right) \quad (1.11)$$

The third boundary condition of (1.11) is obtained from (1.8) while taking account of the inequality $p_0 \geq p_d$.

We recall the properties of the steady-state solutions [3–5]. From (1.10) and the third boundary condition, we obtain

$$C_i = c_{i0}, \quad K \partial_\eta p = q = Q / (2\pi k h) \quad (1.12)$$

The constant of integration Q is the production rate of the well, and h is the productive height of the reservoir.

In accordance with (1.12), the pressure is determined from the ordinary differential equation $p = p(\eta + \alpha, q, c_{i0})$. The free parameters α and q are determined from the first two boundary conditions of (1.11).

Thus, if the first two boundary conditions of (1.11) are not used, it is possible to obtain the steady-state solution for the densities in the functional form

$$n_i = n_i(\eta + \alpha, q, c_{j0})$$

Quantities calculated for the steady solution will subsequently be denoted by an asterisk. We now determine the set of vector fields $e_i^j = e_i^j(\eta)$, where j is the number of the field and i is the number of the component, from the formulae

$$\begin{aligned} e_i^0(\eta) &= \frac{\partial}{\partial \alpha} n_i(\eta + \alpha, Q, c_{j0})|_{\alpha=0} = \left(\frac{\partial n_i}{\partial \eta} \right)_* \\ e_i^\alpha(\eta) &= \left(\frac{\partial n_i}{\partial C_\alpha} \right)_{p^*} = m_i^\alpha - m_j^\alpha \left(\frac{\partial p}{\partial n_j} \right)_* (\partial_\eta p^*)^{-1} e_i^0 \\ m_i^\alpha(\eta) &= \frac{\partial}{\partial c_{\alpha 0}} n_i(\eta, q, c_{j0}) \end{aligned} \quad (1.13)$$

In carrying out the differentiation with respect to the concentrations in formulae (1.13), account is taken of the fact that there is a dependence only on M of the quantities C_α and $c_{\alpha 0}$ by virtue of the normalization equalities $\sum C_i = \sum c_{i0} = 1$.

The vector fields $e_i^j(\eta)$ obviously define a basis in an $(M + 1)$ -dimensional space for each value of η . Any other vector field can be decomposed using this basis. Let $\delta n_i = \delta n_i(t, \eta)$ be small perturbations of the steady solution. We carry out an expansion in the basis

$$\delta n_i(t, \eta) = e_i^j(\eta) x_j(t, \eta)$$

and substitute it into the dynamical equation in the perturbations which follows from (1.10)

$$m \partial_t (r^2 \delta n_i) - k \partial_\eta (\delta K C_i \partial_\eta p + K \delta C_i \partial_\eta p + K C_i \partial_\eta \delta p) = 0$$

We obtain a system of $M + 1$ linear equations in the $M + 1$ unknown functions $x_j(t, \eta)$ which describe the dynamics of the perturbations

$$\begin{aligned} m(kq)^{-1} r^2 G_\alpha^j \partial_t x_j - \partial_\eta x_\alpha &= 0 \\ m(kq)^{-1} r^2 \rho^j \partial_t x_j - \partial_\eta (v^\alpha x_\alpha) - \partial_\eta^2 x_0 &= 0 \end{aligned} \quad (1.14)$$

where the following notation has been used

$$\rho^j = \sum_{i=1}^M e_i^j, \quad v^\alpha = (\partial \ln K / \partial C_\alpha)_{p^*}, \quad G_\alpha^j = e_\alpha^j - c_{\alpha 0} \rho^j$$

The boundary conditions on the functions $x_j(t, \eta)$ are obtained from (1.1) and have the form

$$x_0|_{\eta=0} = 0, \quad x_0|_{\eta=\zeta} = 0, \quad x_\alpha|_{\eta=\zeta} = 0 \quad (1.15)$$

If some particular boundary conditions are specified for the functions $x_j(t, \eta)$ then problem (1.14), (1.15) becomes a closed problem and this enables us to determine the evolution of the perturbations of the steady solution with time. However, in order to investigate the stability in the small, it is not necessary to solve this problem in general form. Separation of the variables in problem (1.14), (1.15) is possible by virtue of the fact that the coefficients in the equations are independent of time. It is sufficient to investigate this particular class of solutions and to elucidate whether there are solutions which do not decay with time. If there are no such solutions the steady filtration flow is stable in the small. This problem is considered in the next section.

2. To analyse the stability we need to use linear mappings in the various functional spaces and we shall now introduce the required definitions and notation.

The scalar product (\cdot, \cdot) and the norm $|\cdot|_I$ in the complex space C^I , where $I \geq 1$, are defined in the usual way

$$(v, u)_I = \sum_{a=1}^I \bar{v}_a u_a, \quad |u|_I = (u, u)_I^{1/2}, \quad u, v \in C^I$$

We recall the expressions for the norms and scalar products in the corresponding spaces of the functions in the range $[0, \zeta]$ with values in C^I the space $L^2_{C^I}[0, \zeta]$

$$(\varphi, \psi)_{L,I} = \int_0^\zeta (\varphi(\eta), \psi(\eta))_I d\eta$$

$$\|\varphi\|_{L,I} = (\varphi, \varphi)_{L,I}^{1/2}, \quad \varphi, \psi \in L^2_{C^I}[0, \zeta]$$

the space $H_{1,C^I}[0, \zeta]$

$$(\varphi, \psi)_{H,I} = (\varphi, \psi)_{L,I} + \zeta^2 (\partial_\eta \varphi, \partial_\eta \psi)_{L,I}$$

$$\|\varphi\|_{H,I} = (\varphi, \varphi)_{H,I}^{1/2}, \quad \varphi, \psi \in H_{1,C^I}[0, \zeta]$$

the space $L^\infty_{C^I}[0, \zeta]$

$$\|\varphi\|_{\infty,I} = \text{ess sup} |\varphi(\eta)|_I, \quad \varphi \in L^\infty_{C^I}[0, \zeta]$$

The norms of the continuous linear mappings, which will be considered later, are induced in the usual way by the norms of the linear spaces in which they act.

In the case of a linear operator B in a space C^I , we shall denote the corresponding adjoint operator by the symbol B^+ . We recall the useful equality

$$|B|_I \leq (\text{Tr}(BB^+))^{1/2} \tag{2.1}$$

In accordance with what has been said in Section 1, we carry out a separation of the variables in problem (1.14), (1.15)

$$x_0(t, \eta) = e^{i\omega t} z(\eta), \quad x_\alpha(t, \eta) = e^{i\omega t} y_\alpha(\eta)$$

Substituting these expressions into relations (1.14) and (1.15) we obtain a system of ordinary differential equations with the following boundary conditions

$$i\omega A y + i\omega v_1 z - \partial_\eta y = 0, \quad (i\omega h_1 + h_2) y + i\omega f_1 z - \partial_\eta^2 z = 0 \tag{2.2}$$

$$y|_{\eta=\zeta} = 0, \quad z|_{\eta=0} = 0, \quad z|_{\eta=\zeta} = 0 \tag{2.3}$$

Here, y is a column vector with components y_α , A is an $M \times M$ matrix with components $A_{\alpha\beta} = m(kq)^{-1} r^2 G_{\alpha\beta}^0$, v_1 is a column vector with components $v_{1\alpha} = m(kq)^{-1} r^2 G_{\alpha}^0$, h_1 is a row vector with components $h_{1\alpha} = (m(kq)^{-1} r^2 \rho^\alpha - v^\beta A_{\beta\alpha})$, h_2 is a row vector with components $h_{2\alpha} = (-\partial_\eta v^\alpha)$ and $f_1 = (m(kq)^{-1} r^2 \rho^0 - v^\beta v_{1\beta})$.

We shall consider a partial problem in the vector function $y(\eta)$ (the first equation of (2.2) and the first boundary condition of (2.3)) together with the complete problem (2.2), (2.3). This partial problem is referred to as problem A .

In addition to the assumptions which have been described in Section 1, we shall adopt the following number of additional assertions

$$(u, Au)_M \geq \lambda(u, u)_M, \quad \lambda > 0, \quad u \in C^M \tag{2.4}$$

$$A = A^+, \quad f_1 > 0 \tag{2.5}$$

We shall now discuss the physical meaning of these conditions. Under the assumption that the total density n depends only slightly on the concentrations C_α at a fixed pressure p , condition (2.4) means that an increase in a certain concentration C_α (or a linear combination of concentrations $\gamma^\alpha C_\alpha$) leads to an increase in the corresponding concentration c_α (or the corresponding linear combination of concentrations $\gamma^\alpha C_\alpha$). The matrix equality in (2.5) can be interpreted such that the effect of a change in the concentration C_α on the concentration c_β is the same as the effect of a change in the concentration C_β on the concentration c_α . When the small quantity $v^\beta v_{1\beta}$ is neglected, the inequality in (2.5) reduces to the assertion that $n_* = n_*(\eta)$ is a monotonically increasing function. Let us put $f_0 = \min f_1(\eta)$. In accordance with (2.5), $f_0 > 0$.

We shall now seek a solution of problem (2.2), (2.3) when $\text{Im } \omega \leq 0$, which corresponds to non-decaying perturbations. We will initially concentrate on problem A and determine the matrix-valued function $U = U(\eta, \zeta)$ as the solution of the following Cauchy problem

$$\partial_\eta U(\eta, \xi) = i\omega A(\eta)U(\eta, \xi), \quad U(\xi, \xi) = 1 \tag{2.6}$$

Lemma 1. The inequality

$$|U(\eta, \xi)|_l \leq \exp(\text{Im } \omega \lambda(\xi - \eta))$$

holds when $\eta \leq \xi$.

Proof. Consider the function

$$F(\eta) = \text{Tr}(U(\eta, \xi)U(\eta, \xi)^+)$$

Using equality (2.6) as well as (2.4) and the equality from (2.5), we obtain

$$\partial_\eta F = -2 \text{Im } \omega \text{Tr}(A U U^+) \geq -2 \text{Im } \omega \lambda F$$

whereupon we derive the inequality $F(\eta) \leq \exp(2 \text{Im } \omega \lambda(\xi - \eta))$ from the well-known Gronwall inequality [11].

On applying inequality (2.1), we obtain the statement of the lemma.

Lemma 2. Let $h_0 = h_0(\eta)$ be a row vector which is a fixed element of the space $L_{CM}^\infty [0, \zeta]$ and let $v_0 = v_0(\eta)$ be a column vector which is an arbitrary element of the space $H_{1,CM} [0, \zeta]$. We now consider a linear mapping from the space $H_{1,CM} [0, \zeta]$ into the space $L_C^2 [0, \zeta]$

$$(L v_0)(\eta) = \int_\eta^\zeta h_0(\eta) U(\eta, \xi) v_0(\xi) d\xi \tag{2.7}$$

This mapping is continuous and its norm satisfies the inequalities

$$\begin{aligned} \|L\| &\leq \theta_1, \quad \|L\| \leq \theta_2 |\omega|^{-1} \\ \theta_1 &= 2^{-1/2} \|h_0\|_{\infty, M} \zeta, \quad \theta_2 = (2 + 2^{1/2} + 3^{-1/2}) \|h_0\|_{\infty, M} \Gamma_0 \\ \Gamma_0 &= \|A^{-1}\|_{\infty, M^2} + \zeta \|\partial_\eta A^{-1}\|_{\infty, M^2} \end{aligned} \tag{2.8}$$

Proof. The operator L is an integral operator and it is therefore continuous as an operator from the space $L_{CM}^2 [0, \zeta]$ into the space $L_C^2 [0, \zeta]$ [9]. The corresponding norm is easily estimated using Lemma 1 and the fact that the norm of an integral operator has an upper limit set by the norm of the kernel in L^2 [9]. By carrying out calculations, we show that the estimate is identical to the number on the right-hand side of the first inequality of (2.8). Next, since the space $H_{1,CM} [0, \zeta]$ is imbedded into the space $L_{CM}^2 [0, \zeta]$ and the norm of the inclusion operator does not exceed unity, we finally obtain the first inequality of (2.8).

By considering problem (2.6), it can be shown that

$$U(\eta, \xi)^{-1} = U(\xi, \eta)$$

whence we obtain the equality

$$\partial_{\xi} U(\eta, \xi) = -i\omega U(\eta, \xi) A(\xi)$$

This enables us to transform expression (2.7) using the formula for integration by parts:

$$(L\nu_0)(\eta) = h_0(\eta)U(\eta, \xi)(-i\omega A(\xi))^{-1}\nu_0(\xi) + h_0(\eta)(i\omega A(\eta))^{-1}\nu_0(\eta) + \int_{\eta}^{\xi} h_0(\eta)U(\eta, \xi)\partial_{\xi}((i\omega A(\xi))^{-1}\nu_0(\xi))d\xi$$

We will now estimate the norm of each term in $L^2_C[0, \zeta]$. Here we shall take account of the fact that $H_{1,C^M}[0, \zeta]$ is imbedded in the space of continuous vector functions [10, 12] and, moreover, that the norm of the inclusion operator does not exceed the number $\zeta^{-1/2}(1 + 3^{-1/2})$. Additionally, we again make use of the estimate for the norm of an integral operator and the result of Lemma 1. The second inequality of (2.8) is then obtained.

The inequality

$$\|L\| \leq 2(\theta_1^{-1} + \theta_2^{-1}|\omega|)^{-1} \tag{2.9}$$

follows from inequalities (2.8).

The solution of problem A is found using the formula

$$y(\eta) = i\omega \int_{\eta}^{\zeta} U(\eta, \xi)\nu_1(\xi)z(\xi)d\xi$$

Using the condition (2.2) we arrive at the following integro-differential equation

$$i\omega(i\omega L_1 + L_2)z + i\omega f_1 z - \partial_{\eta}^2 z = 0 \tag{2.10}$$

$$(L_a z)(\eta) = \int_{\eta}^{\zeta} h_a(\eta)U(\eta, \xi)\nu_1(\xi)z(\xi)d\xi, \quad a = 1, 2$$

Let Δ be the usual self-adjoint extension of the Laplace operator ∂_{η}^2 into the space $L^2_C[0, \zeta]$, which is defined by the second and third boundary conditions of (2.3) [9, 12] and let $\tau = (-\Delta)^{-1/2}$. It is obvious that τ is a continuous linear mapping from $L^2_C[0, \zeta]$ into $H_{1,C}[0, \zeta]$ and, moreover, that $\|\tau\| \leq \zeta(1 + \pi^{-2})^{1/2}$. The quantity τ may also be considered as an operator in $L^2_C[0, \zeta]$. Then, $\|\tau\| \leq \zeta\pi^{-1}$.

We make the substitution $z = \tau Z$ in Eq. (2.10) and we then have a linear equation for Z in the space $L^2_C[0, \zeta]$.

$$Z + TZ = 0 \tag{2.11}$$

$$T = T(\omega) = i\omega V_0(i\omega V_1 + V_2)$$

$$V_0 = (1 + i\omega\tau f_1\tau)^{-1}, \quad V_1 = \tau L_1\tau, \quad V_2 = \tau L_2\tau$$

Theorem. The linear operator T is continuous in the space $L^2_C[0, \zeta]$. Its norm has an upper limit of

$$D = D(\omega) = |\omega| |1 + i\omega\zeta^2\pi^{-2}f_0|^{-1} 2 \left(\frac{|\omega|}{\theta_{11} + \theta_{21}|\omega|} + \frac{1}{\theta_{12} + \theta_{22}|\omega|} \right)$$

$$\theta_{1a}^{-1} = 2^{1/2}\zeta^2\pi^{-1}(1 + \pi^{-2})^{1/2}\|h_a\|_{\infty, M}\Gamma_1$$

$$\theta_{2a}^{-1} = 2(2 + 2^{1/2} + 3^{-1/2})\zeta^2\pi^{-1}(1 + \pi^{-2})^{1/2}\|h_a\|_{\infty, M}\Gamma_1\Gamma_0$$

$$a = 1, 2$$

$$\Gamma_1 = \|v_1\|_{\infty, M} + \zeta\|\partial_{\eta}v_1\|_{\infty, M}$$

The function $D(\omega)$ is bounded in the lower complex half plane. Let D_0 be its maximum value. If

$D_0 < 1$, then Eq. (2.11) only has a zero solution and, consequently, the steady filtration flow is stable.

Proof. According to what has been said above, all of the operators V_a are continuous in the space $L^2_C[0, \zeta]$. An estimate for the norm of the operator T is obtained as the result of the application of inequality (2.9) and the estimates for the norm of the operator τ . If $D_0 < 1$, then the operator T is a contractive operator and Eq. (2.11) only has a zeroth solution.

So, a sufficient condition for stability has been found: $D_0 < 1$. However, it is not a necessary condition. If this inequality is violated, it may nevertheless turn out that Eq. (2.11) does not have non-zero solutions. In the case when $D_0 \geq 1$, the assertions of the theorem can be used to prove stability in a certain range of frequencies which is often sufficient in practice.

3. The stability test which has been found can only be useful in the case when the magnitude of D_0 actually turns out to be sufficiently small for actual filtration flows.

We shall now estimate the order of magnitude of D_0 under the typical working conditions of an operational well in a gas-condensate deposit. We shall confine ourselves to the case of a developed two-phase flow when $p_0 = p_d$ and the condensate occupies a significant part of the pore volume. We shall derive all estimates in the SI system of units.

First, we fix the geometrical parameters of the problem: $r_w = 0.1$ m, $r_0 = 30$ m, $\zeta \approx 5$. Next, we specify the filtration capacity characteristics $m \approx 0.1$, $k \approx 10^{-15}$ m² and the pressure conditions: reservoir pressure $p_0 \approx 5 \times 10^7$ Pa and the pressure drop $\Delta p = (p_0 - p_w) \approx 5 \times 10^6$ Pa.

The quantities n_g and n_c may be of the same order of magnitude but differ several-fold:

$$n_g \approx n_c \approx \Delta n \approx 10^4 \text{ mol m}^{-3}, \quad \Delta n = n_g - n_c \quad (3.1)$$

The typical orders of magnitude of the viscosities are as follows:

$$\mu_g \approx 10^{-5} \text{ Pa s}, \quad \mu_c \approx 10^{-4} \text{ Pa s} \quad (3.2)$$

It can be seen from (3.1) and (3.2) that the quantity K can take values of the order of 10^8 mole (J s⁻¹) or 10^9 mole (J s⁻¹). We use the lower value as the estimate and then find the quantity

$$q \approx k \zeta^{-1} \Delta p \approx 10^{14} \text{ mol m}^{-3} \text{ s}^{-1} \quad (3.3)$$

We now consider the quantity $\partial_\eta n$. The total density n changes considerably on account of the pronounced differences in the distribution of the phases: close to the well the pore space is mainly filled with the liquid phase while, away from the well, it is mainly filled with the gas phase. It is therefore possible to put $\partial_\eta n = \zeta^{-1} \Delta n$ (see (3.1)). Now, when account is taken of (3.3), it is easy to estimate the magnitude of f_1 and, consequently, the magnitude of f_0

$$f_0 \approx f_1 \approx n(kq)^{-1} r^2 \zeta^{-1} \Delta n \approx 2 \times 10^5 \text{ s} \quad (3.4)$$

We shall neglect the quantities ρ^α as well as the derivatives $\partial_\eta A$, $\partial_\eta v_1$, $\partial_\eta v^\alpha$.

We now consider the matrix A . It is actually calculated using the matrix $(\partial c_\alpha / \partial C_\beta)_p$. Calculations using semi-empirical equations show that the off-diagonal elements of the latter matrix are negligibly small while the diagonal elements are of the order of 10^{-1} . We obtain

$$\lambda \approx 10^5 \text{ s}, \quad \|A\|_{\infty, M^2} \approx M\lambda, \quad \Gamma_0 \approx \|A^{-1}\|_{\infty, M^2} \approx M\lambda^{-1} \quad (3.5)$$

The vector v_1 is estimated in the same way as the quantity $\partial_\eta n$. As a result, we obtain

$$\Gamma_1 \approx \|v_1\|_{\infty, M} \approx 10^3 \text{ s} \quad (3.6)$$

We shall study the quantities v^α . At a fixed pressure, the function K can vary strongly when the vector δC_α is collinear with $(C_\alpha - c_\alpha)$. However, the relative change is very small. Calculations show that $\|v\|_{\infty, M} \approx 10^{-3}$. It is important that the estimate is independent of M .

When account is taken of (3.5), we now obtain $\|h\|_{\infty, M} \approx 10^{-3} M\lambda$. On combining this result with (3.4)–(3.6), we find that the approximate function

$$D_1(\omega) = \alpha_1 |\omega|^2 |1 + i\omega\alpha_2|^{-1} (1 + \alpha_3 |\omega|)^{-1} \\ \alpha_1 = 10^7 \text{ m s}^2, \quad \alpha_2 = 2 \times 10^7 \text{ s}, \quad \alpha_3 = M^{-1} 10^5 \text{ s}$$

can be used instead of the exact function $D(\omega)$.

The stability condition is obviously satisfied when $M \leq 10$. It may be pointed out that the behaviour of the function $D_1(\omega)$ deteriorates as M increases. This does not mean that an increase in the number of components in the mixture destroys stability. It simply means that when a large number of components are taken into account it is necessary to use finer integral estimates which would be "sensitive" to the extent of the effect of a component on the two-phase equilibrium. The stability test which has been obtained above is well adapted to a system in which all the components have an approximately equal effect on the two-phase state.

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